Mathematical Properties of Tellings

Joshua Gerrish

February 16, 2001

1 About Tellings

Tellings is a fantasy RPG designed by Ed Chang. To play the game, 10 sided dice are used. As a common shorthand notation, we will use 2d10 and 1d10 to refer to a roll of two dice and a roll of 1 die respectively. There are three basic rolls used in the game:

- The first is the roll or fail. 2d10 is rolled, and the sum must be at or below the target.
- The second is the percentile roll. In this roll, 2d10 is rolled, and one die is assigned the 10 digits, and the other is assigned the ones.
- The last roll is the roll or base. In this roll, 1d10 is rolled, and the if the result is at or below the base, the result of the roll is used as the end value. Otherwise, the base is used as the end value.

2 Basics of Probability

Before talking about the nature of probability in Tellings, some basics of probability must be covered.

Two events are independent if the result of one does not affect the outcome of the other. Dice rolling is a classic example of independent events. If you roll two dice, the outcome of one does not affect the outcome of the other.

The probability of two independent events, A and B, both occuring is $P(A \text{ and } B) = P(A) \times P(B)$. So for example, the probability of rolling double ones with 2d10 is $\frac{1}{10} \times \frac{1}{10} = \frac{1}{100}$.

If A and B are not mutually exclusive, then the probability of A or B is:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Here, when $P(A)$ and $P(B)$ are added, the union of $P(A)$ and $P(B)$ is counted twice, so we must subtract this union. For example, what is the probability of rolling a 1 at least once? In this case the probability is $\frac{1}{10} + \frac{1}{10} - \frac{1}{100} = \frac{19}{100}$.

Discrete probability is closely related to a field of mathematics called combinatorics. Combinatorics deals with combinations and arrangements of objects.

Roll	1	2	3	4	5	6	7	8	9	0
1	2	3	4	5	6	7	8	9	10	11
2	3	4	5	6	7	8	9	10	11	12
3	4	5	6	7	8	9	10	11	12	13
4	5	6	7	8	9	10	11	12	13	14
5	6	7	8	9	10	11	12	13	14	15
6	7	8	9	10	11	12	13	14	15	16
7	8	9	10	11	12	13	14	15	16	17
8	9	10	11	12	13	14	15	16	17	18
9	10	11	12	13	14	15	16	17	18	19
0	11	12	13	14	15	16	17	18	19	20

Figure 1: Dice Combinations

For example, how many ways are there to choose two items from a set of three items? Well, you could chose the first and second. Or you could choose the first and third, or you could choose the second and third. In this case there are three ways. The notation used to describe choosing r items from a set of n is:

$$\binom{n}{r}$$

Read n choose r.

With n trials, how many successes are we likely to have? The answer is given by the binomial distribution, and the average is np. The variance is then npq.

The average wait time is based on the mean of the negative binomial. This mean is $rq \div p$. Where r is the number of successes we are waiting for, p is the probability of success and q is the probability of failure.

More to follow...

3 Probabilities in Tellings

Because two 10 sided dice are used in tellings, the number of possible combinations is 100. This value is found by multiplying 10 by 10.

3.1 Roll or Fail

In a roll or fail roll, the possible values are in the range 2 through 20. How many possible ways are there to roll 2? Only one, because the only way to get 2 is by rolling two ones. How about the number of ways to roll a 3? There are two possible ways this time. If the first die is a one, and the second die is a two, or if the first die is a two and the second die is a one. But what about higher numbers? Figure **??** shows all possible combinations.

	1	2	3	...	100
+	100	99	98	...	1
	101	101	101	...	101

You can notice that the possible ways increases by one, up until a roll of 11, after which it decreases by one. This is essentially an inverse absolute value function. To find the percentage of any roll on 2d20, use this formula:

$$P(n) = \frac{10 - |11 - n|}{100} \tag{1}$$

Unfortunately, for most rolls in Tellings, what is wanted is not the exact dice roll, but instead that roll or less. How is this value found? One method is by summing all probabilities up to and including the intended one.

$$\sum_{i=2}^{n} P\left(i\right)$$

Where $P\left(i\right)$ is equation 1 and n is the target roll. Another method is by approximating using a normal distribution. Let's assume we want to use the first method. Chart **??** shows this. These numbers are also called the triangular numbers or bowling pin numbers. The reason they are named this is beacause they form a triangle.

But we can find an exact value for this, without using summation.

The problem of finding the sum of consecutive integers is an ancient problem, and there is a classic mathematical story that is behind it. The story involves Karl Friedrich Gauss, who is perhaps the most famous mathematician in the world. His elementary school teacher wanted to keep the class busy, so she asked them to sum the numbers from 1 to 100. Gauss came up with the answer in less than a minute, and it was correct. How did he solve it? He noticed that by aligning the numbers and their reverses, then summing these, the values would be the exact same.

The sum is then:

$$\sum_{i=1}^{n} i = \frac{n\left(n+1\right)}{2}$$

We can use this for our own formula by noting it is symmetric about eleven.

$$\sum_{i=2}^{n} P\left(i\right) = \begin{cases} \frac{(n-1)n}{2} & \text{if } n \leq 11, \\ 100 - \frac{21-n(20-n)}{2} & \text{if } x > 11. \end{cases} \tag{2}$$

So now we have a method of determining the exact probability of rolling at or below a certain number. There are several other interesting problems. What if we want to know how many successes we will have if we roll a certain number of times? For example, in five rolls, what is our chance of getting five successes?

Dice Total	Probability	Sum Probability
2	1	1
3	2	3
4	3	6
5	4	10
6	5	15
7	6	21
8	7	28
9	8	36
10	9	45
11	10	55
12	9	64
13	8	72
14	7	79
15	6	85
16	5	90
17	4	94
18	3	97
19	2	99
20	1	100

Figure 2: Totals of dice throws

Four? Three? etc.? Or, if a certain roll is unlikely to succeed, how many times would we need to roll it before our chances improve to an acceptable level?

We can consider each roll an independent event, either succeeding or failing with probability p. This is then a binomial distribution, and the mean value is the number of rolls n times p.

The mean value of the first success is then $\frac{q}{p}$.

3.2 Example 1

A player needs to roll an 8 or less to successfully hit a creature. By equation **??** his success probability is thus

$$\frac{\frac{7\times 8}{2}}{100} = \frac{28}{100}$$

The average number of rolls before a success is then

$$\frac{72}{28} = \frac{18}{7} = 2.5714$$

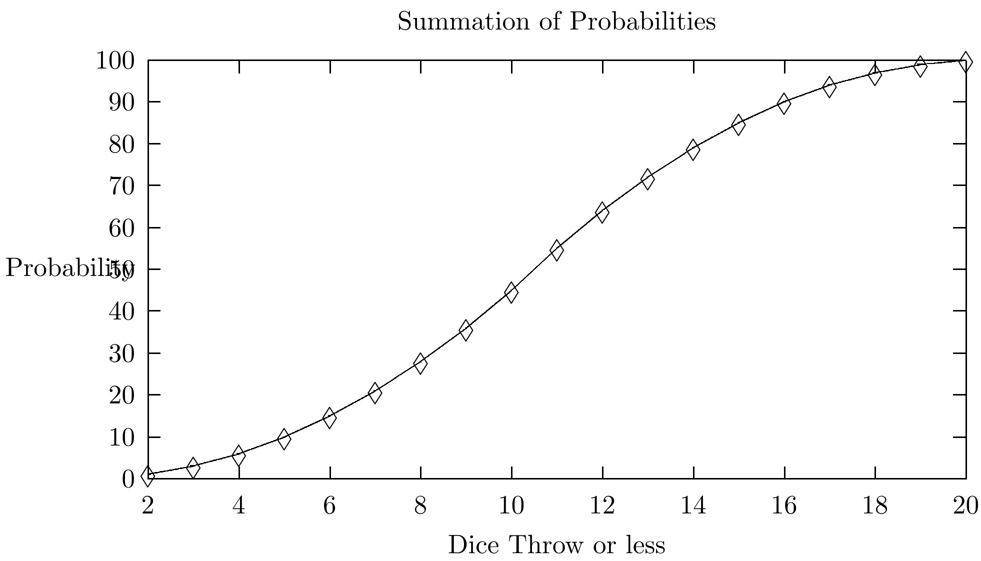

Figure 3: Base or Fail rolls

3.3 Percentile Roll

The percentile roll has a simple statistical behavior. There is a one to one correspondance. If you need to roll n% or below, then your probability is simply n%.

3.4 Roll or Base

In the roll or base, wre are interested in the average value of a roll as a function of m, the original base, and n, the modified base value. For values of $n \geq 10$ the result is simply the average of a 1d10 roll, or 5.5. For all other values, the mean value can be found by this formula:

$$\frac{\sum_{i=1}^{n} i + m(10-n)}{10} \tag{3}$$

This can be reduced to:

$$\frac{\frac{n(n+1)}{2} + m(10-n)}{10} \tag{4}$$

For the case when $m = n$ this can be further reduced to:

$$\frac{n(21-n)}{20} \tag{5}$$

Figure **??** shows the behavior of the Roll or Base roll.

3.5 Example 2

A player has a base initiative of 4. His dagger adds a 1 to this to make his init value 5. What is his average initiative going to be?

$$\frac{\frac{5\times 6}{2} + 4(10-5)}{10} = \frac{15+20}{10} = 3.5$$

This doesn't mean the player will on average roll a 4 on his dice. What it means is the final result will tend towards 4.

4 Conclusion

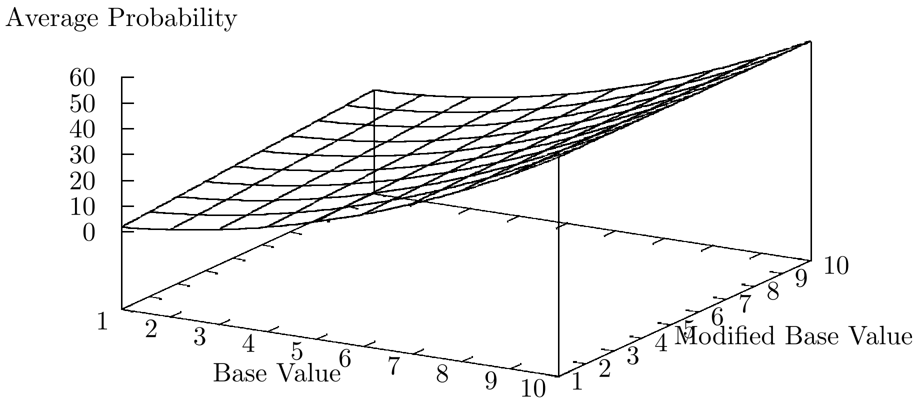

Figure 4: Roll or Base average result